(Due Tuesday 04/02/2019 right before the class)

&

(Your homework shall be stapled if it contains multiple pages.)

SPRING/2019/MA526: HOMEWORK 7

Instructor: Guangqu Zheng[1]; Grader: Chessa Mccalla[2]

Total points: 20

Q1 (3 pt) A standard exponential random variable X with parameter 1, has the following density function

$$f(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ e^{-x} & \text{if } x > 0 \end{cases}$$

Put $Y = \sqrt{X}$, find the probability density function of Y.

Q2 (2pt) Look up the normal table and find the value k such that

$$\mathbb{P}\big(-1.44 < Z < k\big) = 0.24466\,,$$

where $Z \sim \mathcal{N}(0,1)$ is a standard normal random variable.

Q3 (4pt) Let X be a lognormal$(0, 100)$ random variable, that is, X is a positive random variable and $\log_{10} X$ is a normal random variable with mean 0 and standard deviation 10. Denote by F the cumulative distributional function of X, then find the following values:

$$F(1) \qquad \textit{and} \qquad F(10)\,.$$

It is enough for you to express the above values in term of the standard normal CDF Φ. If you find their approximate value using the normal table, it will be nice.

Q4 (3pt) In an urn that contains 3 green balls, 2 blue balls and 5 red balls, we randomly sample 5 balls. What is the probability that both blue balls and at least 1 red ball are selected?

Q5 (4pt) There are two basketball teams A, B and from the data, we assume that A wins B for a generic match with probability 3/4.

(1) If A, B are in the final and one has to win the championship by winning 4 out of 7, so what is the probability that A gets the championship on the fifth game?

(2) If in the next season, we assume that A, B will face each other for 7 games. What is the probability that A wins 5 out of 7 games?

Hint: be careful....

Q6 (4pt) Let X be a discrete uniform random variable over $\{1, 2, \ldots, 6\}$ and let Y be a Bernoulli random variable with parameter 1/2 such that X, Y are independent.

(1) Find the PMF of the random variable Z, where $Z = XY$.

(2) Compute the third moment of Z, that is, $\mathbb{E}\big[Z^3\big]$.

[1] gzheng90@ku.edu; Office hours: TuTh 11:00-11:50; Office = 641 Snow Hall

[2] chessa_m@ku.edu