

(Due Tuesday 03/19/2019 right before the class)

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(Your homework shall be stapled if it contains multiple pages.)

SPRING/2019/MA526: HOMEWORK 5

Instructor: Guangqu Zheng¹; Grader: Chessa Mccalla²

Total points: 20

Q1 (2 + 2 + 2 points) Find the exact value of k such that

$$f(x) = k \binom{3}{x} \frac{1}{5^x} (4/5)^{3-x}, \quad x = 0, 1, 2, 3$$

defines a probability mass function. An approximate value of k from your calculator can not be regarded as a right answer.

Find the mean and variance of a random variable X if X has the above probability mass function.

Q2 (2 + 2 + 2 points) If the joint probability distribution of X and Y is given by

$$f(x, y) = Cx(1 + 2y^2) \quad \text{for } 0 < x < 2 \quad \& \quad 0 < y < 1; f(x, y) = 0, \text{ elsewhere.}$$

- (a) Find the exact value of C .
- (b) Compute $\mathbb{E}[XY^2]$
- (c) Find the variance of XY^2 .

Q3 (2 + 2 points) Find the covariance of two random variables X, Y that have the following joint density function

$$f(x, y) = \begin{cases} x + y & x, y \in (0, 1) \\ 0 & \text{elsewhere.} \end{cases}$$

Find the probability that $X + Y > 1/2$.

Q4 (2 + 2 points)

The density function of the random variable X is

$$f(x) = \begin{cases} 6x(1 - x), & 0 < x < 1 \\ 0, & \text{elsewhere.} \end{cases}$$

Let μ, σ denote its mean and standard deviation respectively.

- (a) compute $\mathbb{P}(\mu - 2\sigma < X < \mu + 2\sigma)$.
- (b) Compare the result (a) with the bound obtained from Chebyshev's theorem.

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