(Due Tuesday 02/26/2019 right before the class)

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(Your homework shall be stapled if it contains multiple pages.)

SPRING/2019/MA526: HOMEWORK 4

Instructor: Guangqu Zheng[1]; Grader: Chessa Mccalla[2]

Total points: 20

Q1 (2+2 points) In the lecture, we know how to obtain a CDF from a PDF: If $f(x)$ is the probability density function of a continuous random variable X, then the corresponding cumulative distribution function F is given by

$$F(x) = \mathbb{P}\big(X \leq x\big) = \int_{-\infty}^{x} f(y)\, dy\,, \quad -\infty < x < \infty\,.$$

In view of this definition, it is also clear how one can obtain the PDF from a given CDF: Just differential it! To be more precise,

$$f(x) = \frac{d}{dx} F(x)\,.$$

Here is the question 1: The waiting time, in hours, between successive arrivals of customers in the drive-through counter, is a continuous random variable with cumulative distribution function

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 - e^{-7x}\,, & \text{if} \quad x \geq 0 \end{cases}$$

(1) Find the corresponding probability density function f. (2) Find the probability that the waiting time is more than 75minutes.

Q2 (1+2+2+2 points) If the joint probability distribution of X and Y is given by

$$f(x,y) = \frac{x+y}{30} \quad \text{for } x = 0,1,2,3 \quad \& \quad y = 0,1,2,$$

(a) Verify that f is indeed a joint probability mass function.

(b) Find the probability that X is ***strictly bigger*** than Y.

(c) Find the probability that the sum of X and Y is three.

(d) Are X and Y independent? Explain your answer with enough details.

Q3 (2 + 2 points)

The proportion of the budget for a certain type of company $\mathscr{X}$ that is allotted to environmental and pollution control is coming under scrutiny. A data collection project determines that the distribution of these *proportions* is given by

$$f(y) = \begin{cases} 4(1-y)^3\,, & 0 \leq y \leq 1 \\ 0, \text{elsewhere.} \end{cases}$$

(a) Verify that the above is a valid density function.

(b) What is the probability that a company chosen at random expends less than 10% of its budget on environmental and pollution controls?

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Q4 $(1 + 2 + 2$ points)

The joint density function of the random variables X and Y is

$$f(x, y) = \begin{cases} 6x, & 0 < x < 1 \quad \& \quad 0 < y < 1 - x, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Verify f is indeed a joint probability density function.

(b) Show that X and Y are not independent.

(c) Find $\mathbb{P}(X > 0.3|Y = 0.5)$.