solution to hw12 questions, MA526

`Exercises 9.2, 9.6, 9.10 and 9.14 in our textbook`

Q1: The population distribution is (approximately) normal with standard deviation $\sigma = 40$ (hours). The sample size $n = 30$ and it gives us the sample average $\overline{x} = 780$ (hours) and we are looking for the 96% confidence interval. That is, $\alpha = 4\%$ and we find in the normal table that $z_{0.02}$ is between 2.05 and 2.06. `For the rough estimate, you can choose any value in between.`

Review: Suppose Φ is the standard normal CDF, z_α is the number that satisfies $\Phi(z_\alpha) = 1-\alpha$.

The required confidence interval is given by

$$\overline{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} < \mu < \overline{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

with $\overline{x} = 780$, $z_{\alpha/2} = 2.05$, $\sigma = 40$, $n = 30$, that is,

$$765.03 < \mu < 794.97\,.$$

Q2: As in **Q1,** with 96% confidence, the error $|\mu - \overline{x}|$ will be within 10 hours provided the sample size n should be bigger than

$$\left(\frac{z_{\alpha/2}\sigma}{10}\right)^2$$

here $z_{\alpha/2} = z_{0.02} = 2.05$, $\sigma = 40$, so n should be bigger than 67.24. n should be an integer, so we should ask for a **sample size bigger than or equal to 68.**

Q3: $n = 12$, the sample average $\overline{x} = 79.3$, sample standard deviation $s = 7.8$. The population distribution is assumed to be normal but with unknown variance, mean. We are asked to estimate the mean and find the 95% confidence interval. We shall use the t-distribution with degree of freedom 11!

From the t-value table, $t_{0.025} = 2.201$. The desired confidence interval is given by

$$79.3 \pm 2.201 \times \frac{7.8}{\sqrt{12}} = 79.3 \pm 4.96$$

that is, $(74.34, 84.26)$

Q4: There are 15 observation already. We can compute its sample average, which is equal to

$$\overline{x} := \frac{1}{15}\big(3.4 + 2.5 + 4.8 + 2.9 + 3.6 + 2.8 + 3.3 + 5.6 + 3.7 + 2.8 + 4.4 + 4.0 + 5.2 + 3.0 + 4.8\big) \approx 3.79$$

and we can also compute the sample variance using the formula

$$s^2 = \left[\frac{1}{n-1}\sum_{i=1}^{n}(x_i)^2\right] - \frac{n}{n-1}\left(\overline{x}\right)^2$$

in this problem, $s^2 \approx 0.92$, so $s \approx 0.96$. We are asked to find the 95% prediction interval for the next trial: As we do not know σ, we use the t-distribution with degree of freedom 14 and we find in the t-table that

$$t_{0.025} = 2.145$$

so the 95% prediction interval is given by

$$3.79 \pm 2.145 \times 0.96 \times \sqrt{1 + \frac{1}{15}} \quad \text{that is} \quad (1.66, 5.91)$$