solution to hw11 questions, MA526

Q1: χ^2_α here denotes the number that satisfy

$$P(X^2 > \chi^2_\alpha) = \alpha$$

where X^2 is the chi-squared random variable with degree of freedom ν.

(a) $\nu = 5$, $P(X^2 > 16.750) = 0.005$, so $\chi^2_{0.005} = 16.750$.

(b) $\nu = 19$, $P(X^2 > 30.144) = 0.05$, so $\chi^2_{0.05} = 30.144$.

(c) $\nu = 12$, $P(X^2 > 26.217) = 0.01$, so $\chi^2_{0.01} = 26.217$.

Q2: Given X^2 a chi-squared random variable with degree of freedom ν.

(a) $\nu = 21$, $P(X^2 > 38.932) = 0.01$.

(b) $\nu = 6$, $P(X^2 > 12.592) = 0.05$.

(c) $\nu = 10$, we get $P(X^2 > 23.209) = 0.01$ so

$$P(\chi^2_\alpha < X^2 < 23.209) = P(\chi^2_\alpha < X^2) - P(X^2 > 23.209)$$

gives us $P(\chi^2_\alpha < X^2) = 0.025$, so $\chi^2_\alpha = 20.483$.

Q3: The population distribution is approximately $\mathcal{N}(74, 8)$. Then the statistics $(n-1)S^2/\sigma^2$ is approximately a chi-squared random variable with degree of freedom $n-1$. In this particular exercise, $n = 20$, σ^2 is assumed to be 8. Now we have a random sample with $s^2 = 20$, then

$$\frac{(n-1)s^2}{\sigma^2} = 19 \times 20/8 = 47.5.$$

At the first sight, the sample variance $s^2 = 20$ is much bigger than the assumed population variance σ^2, while for a chi-squared random variable X^2 with degree of freedom being 19, we have

$$P(X^2 > 36.191) = 0.01$$

[Students can choose other possible values for their arguments.]

That is, with only 1% chance, one would get a sample variance bigger than 36.191, under the assumption that $\sigma^2 = 8$. So we shall not consider $\sigma^2 = 8$ to be the valid assumption.

Q4: Recall that once we fix the degree of freedom ν, t_α will denote the number satisfying

$$P(T > t_\alpha) = \alpha$$

with $T \sim \texttt{t-distribution}(\nu)$.

(a) Fix $\nu = 20$, $P(T > t_0.01) = 0.01$ and $P(T > t_0.005) = 0.005$. By symmetry of the t-distribution, we get $P(T > -t_{0.005}) = 0.995$. Thus,

$$P(-t_{0.005} < T < t_{0.01}) = P(-t_{0.005} < T) - P(T > t_{0.01}) = 0.995 - 0.01 = 0.985.$$

(b) Assume $T \sim \texttt{t-distribution}(\nu)$, then by definition, we get $P(T > t_{0.025}) = 0.025$, and by symmetry, we get also $P(T < -t_{0.025}) = 0.025$. It follows that

$$P(T > -t_{0.025}) = 1 - 0.025 = 0.975.$$