

FALL/2018/MA526: REVIEW EXERCISES FOR MID-TERM I

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Total points: 0.

Q1 Given the following data

2.0	3.0	0.3	3.3	1.3	0.4
0.2	6.0	5.5	6.5	0.2	2.3
1.5	4.0	5.9	1.8	4.7	0.7
4.5	0.3	1.5	0.5	2.5	5.0
1.0	6.0	5.6	6.0	1.2	0.2

(1) Find the sample average. (2) Find the sample median. (3) Find the sample variance for the numbers in the first row.

Q2 The probability that a patient recovers from a complicated heart operation is 0.4. What is the probability that exactly two of the next three patients who have this operation survive?

Q3 There is a 50-50 chance that the queen carries the gene of hemophilia. If she is a carrier, then each prince has a 50-50 chance of having hemophilia independently. If the queen is not a carrier, the prince will not have the disease. Suppose the queen has had three princes without the disease. What is the probability the queen is a carrier ?

Q4 Let us flip a fair coin consecutively according to the following rule: You continue to toss the coin until you get your first head. Denote by X the number of flipping you need for getting your first head.

- (1) Specify the sample space Ω .
- (2) Find the probability mass function $f_X(x)$ of the random variable X .
- (3) Find the probability that you need at most 3 tossings.

Q5 In how many ways can 4 boys and 5 girls sit in a row if the boys and girls must alternate?

Q6 Dice A favors 6 with probability $3/8$ and other values with equal probability and Dice B is fair. So what is the probability of getting a total of 8?
what is the probability of getting at most a total of 3?

Q7 A real estate agent has 9 keys to open 8 new homes: only 1 of the 9 keys is a master key that can open any home while each of the other 8 keys can only open one home. If 50% of these homes are usually left unlocked, what is the probability that the real estate agent can get into a specific home if the agent selects 3 keys at random before leaving the office?

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Q8 Given the following CDF, find the corresponding pmf.

$$F(x) = \begin{cases} 0, & \text{for } x < 0, \\ \frac{1}{16}, & \text{for } 0 \leq x < 1, \\ \frac{5}{16}, & \text{for } 1 \leq x < 2, \\ \frac{11}{16}, & \text{for } 2 \leq x < 3, \\ \frac{15}{16}, & \text{for } 3 \leq x < 4, \\ 1 & \text{for } x \geq 4. \end{cases}$$

Q9 Determine the value C so that each of the following functions can serve as a probability distribution of the discrete random variable X :

- (a) $f(x) = c(x^2 + 4)$ for $x = 0, 1, 2, 3$;
 (b) $f(x) = c \binom{2}{x} \binom{3}{3-x}$ for $x = 0, 1, 2$.

Q10 Consider the probability density function of X : $f(x) = k\sqrt{x}$ for $x \in (0, 1)$ and $f(x) = 0$ elsewhere. First find the value of k , then find the CDF and use it to evaluate $\mathbb{P}(0.3 < X < 0.6)$.

Q11 If the joint probability distribution of X and Y is given by $f(x, y) = K(x + y)$ for $x = 0, 1, 2, 3$ and $y = 0, 1, 2$. First find the value of K . Then compute the following probabilities:

$$\mathbb{P}(X \leq 2, Y = 1), \mathbb{P}(X > 2, Y \leq 1), \mathbb{P}(X > Y) \text{ and } \mathbb{P}(X + Y = 4).$$

Q12 From a sack of fruit containing 3 oranges, 2 apples, and 3 bananas, a random sample of 4 pieces of fruit is selected. If X is the number of oranges and Y is the number of apples in the sample, find

- (a) the joint probability distribution of X and Y ;
 (b) $\mathbb{P}[(X, Y) \in A]$, where A is the region that is given by $\{(x, y) | x + y \leq 2\}$.
 (c) Find the expected value of $X - Y$.

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Q99 The joint density function of the random variables X and Y is

$$f(x, y) = \begin{cases} K \cdot x, & 0 < x < 1, 0 < y < 1 - x \\ 0 & \text{elsewhere.} \end{cases}$$

- (a) Find the value of K .
 (b) Show that X and Y are not independent.
 (c) Find $\mathbb{P}(X > 0.3 | Y = 0.5)$.
 (d) Find the expected value of $XY + Y^2$.
 (e) Find the variance of X . Then deduce the variance of $10X + 10000000000000$.