

(Due Wednesday 09/05/2018 right before class)

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(Your homework shall be stapled if it contains multiple pages.)

FALL/2018/MA526: HOMEWORK 2

Instructor: Guangqu Zheng¹; Grader: Chessa Mccalla²

Total points: 20. This homework focus on (i) counting (ii) basic set operations (iii) the concept of probability and its additive rules.

Q1 (3 points) List the elements of each of the following sample spaces:

- (a) the set of integers between 1 and 10.89999.
- (b) the set $S = \{x : 2x - 2 < 10 \text{ and } x + 1 > 4\}$.
- (c) the set of outcomes when a coin is tossed until a tail or three heads appear.

Q2 (3 points) Let the sample space Ω be $\{1, 2, \dots\}$ and let $p_i = 2^{-i}$ for each integer $i \geq 1$. Verify that $\{p_i, i \geq 1\}$ defines a sequence of probabilities on Ω .

Q3 (5 points) Recall the De-Morgan's law: Given three sets $A, B, C \subset \Omega$, one has

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \quad \text{and} \quad A \cup (B \cap C) = (A \cup B) \cap (A \cup C).$$

Assume \mathbb{P} is a probability on the sample space Ω , then use the above De-Morgan's law to show

$$\mathbb{P}(A \cup B \cup C) = \mathbb{P}(A) + \mathbb{P}(B) + \mathbb{P}(C) - \mathbb{P}(A \cap B) - \mathbb{P}(A \cap C) - \mathbb{P}(B \cap C) + \mathbb{P}(A \cap B \cap C).$$

Note for students: You are not forced to use De-Morgan's law, it is just a hint. You may be able to find other ways to prove it.

Q4 (3 + 2 points) Let Ω be a sample space, equipped with a probability \mathbb{P} . Suppose A, B are two events with strictly positive probability.

- (1) If we define $Q(C) = \mathbb{P}(C|A)$ for any $C \subset \Omega$. Show that Q is also a probability on Ω .
- (2) If $\mathbb{P}(B|A) = \mathbb{P}(B)$, prove $\mathbb{P}(A|B) = \mathbb{P}(A)$.

Q5 (2+2 points) (a) How many distinct permutations can be made from the letters of the word **INFINITY**?

(b) How many ways are there that no two students will have the same birth date in a class of size 45? (Let us assume that there are 365 days in a year.)

Note for students: we only consider the month and day as the birth date, for example 01/26/1990 and 01/26/1909 are identified as the same birth date.

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